Commission Machines Tournantes

Vibrations asynchrones
Cas des excitations paramétriques

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Outline

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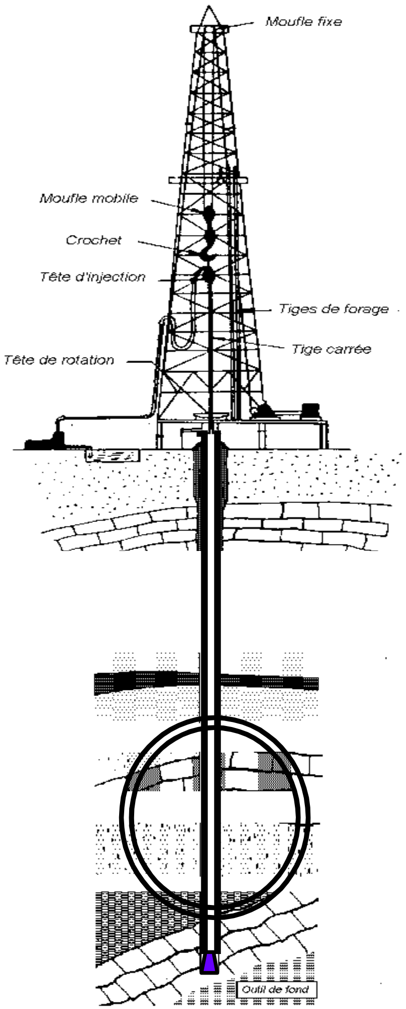
I – Examples. Example 1



Oil drilling
Rotary drilling
(Total – ElF)

Excitation sources

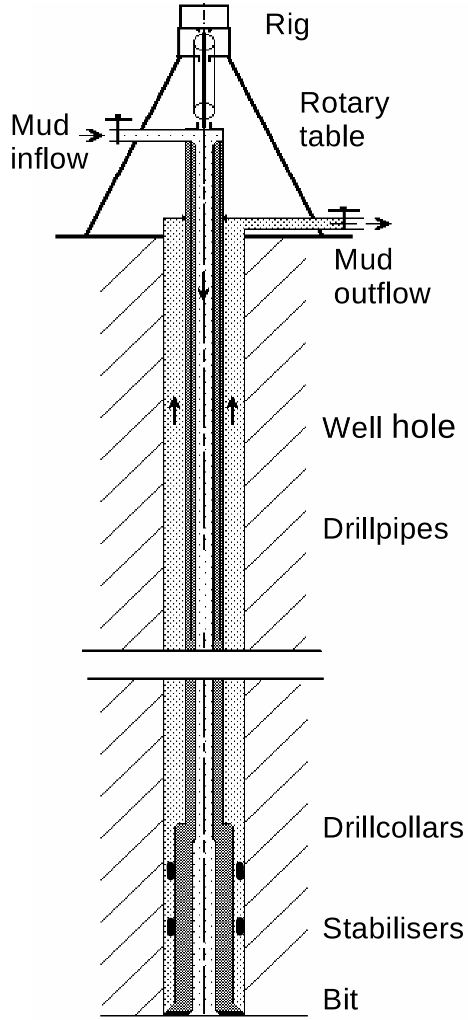
- Bit, (axial force & torque)
- Unbalance masses
- Pulsation of the mud

Observed phenomena

- Forward & backwhirl
- Lateral instability
- Bit bouncing
- Stick slip

Results

- Unscrewed drillpipe
- Wear, fatigue
- Non-controlled drilling
- Speed drilling decreases
- MTBF increases



Example 2.

Helicopter shaft subjected to pulsating torque

II- Modelling

The dynamic behavior a MDOF mechanical system having time-varying parameters is governed by the following set of Mathieu-Hill equation:

$$\mathbf{M}(\Omega)\,\ddot{\mathbf{U}} + \mathbf{C}(\Omega)\,\dot{\mathbf{U}} + \mathbf{K}(\Omega)\,\mathbf{U} = \mathbf{F}(\Omega) \qquad (1)$$

$\mathbf{M}, \mathbf{C}, \mathbf{K}$: Mass, Damping (and/or Gyroscopic), stiffness matrices

$\mathbf{U}, \mathbf{F}$, displacement and force vectors

Ω : forcing angular frequency

Dynamic instability occur when forcing frequency :

$$\Omega \approx \frac{\omega_i \pm \omega_j}{k} \qquad (2)$$

ω_i and ω_j, natural frequencies of the mechanical system

k, an integer, order of the instability.

Longitudinal-lateral and torsion-lateral couplings

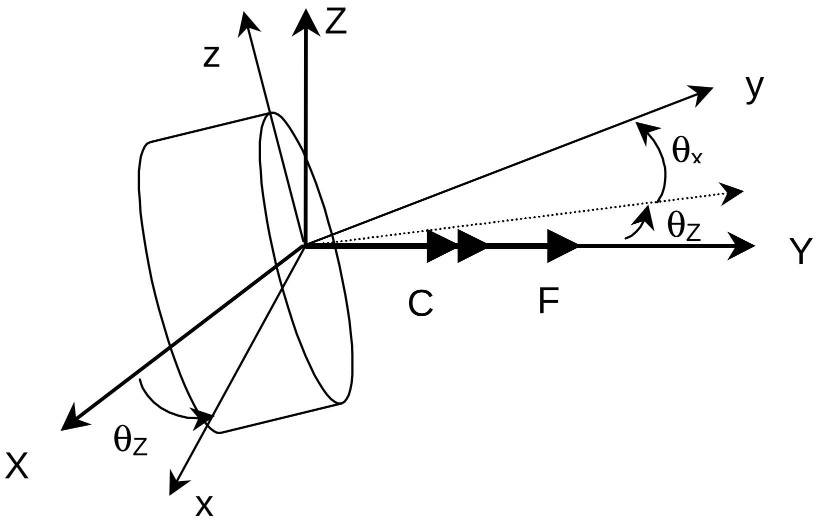


Figure 1

Axial force

$$\vec{F} = F\theta_Z \vec{i} + F\vec{j} - F\theta_x \vec{k} \quad (3)$$

Axial torque

$$\vec{C} = C\theta_Z \vec{i} + C\vec{j} - C\theta_x \vec{k} \quad (4)$$

Lateral forces

$$F\theta_z, -F\theta_x \quad (5)$$

Additional bending moment

$$C\theta_z, -C\theta_x \quad (6)$$

$$\begin{cases} F_X = -T_X + F\theta_Z \\ F_Z = -T_Z - F\theta_x \end{cases} \quad (7)$$

$$\begin{cases} M_X = -EI\dfrac{\partial \theta_X}{\partial y} + C\theta_Z \\ M_Z = -EI\dfrac{\partial \theta_Z}{\partial y} - C\theta_x \end{cases} \quad (8)$$

Equations of motion by using Newton's laws

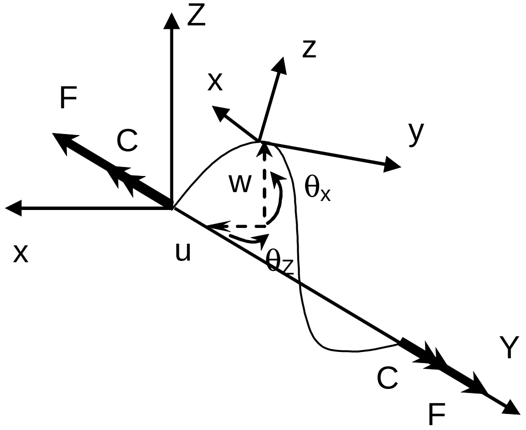


Figure 2

$$\rho S\frac{\partial^2 w}{\partial t^2}+EI\frac{\partial^4 w}{\partial y^4}-F\frac{\partial^2 w}{\partial y^2}+C\frac{\partial^3 u}{\partial y^3}=0 \qquad (9)$$

$$\rho S\frac{\partial^2 u}{\partial t^2}+EI\frac{\partial^4 u}{\partial y^4}-F\frac{\partial^2 u}{\partial y^2}-C\frac{\partial^3 w}{\partial y^3}=0 \qquad (10)$$

Equations of motion by using Lagrange equations and the FEM

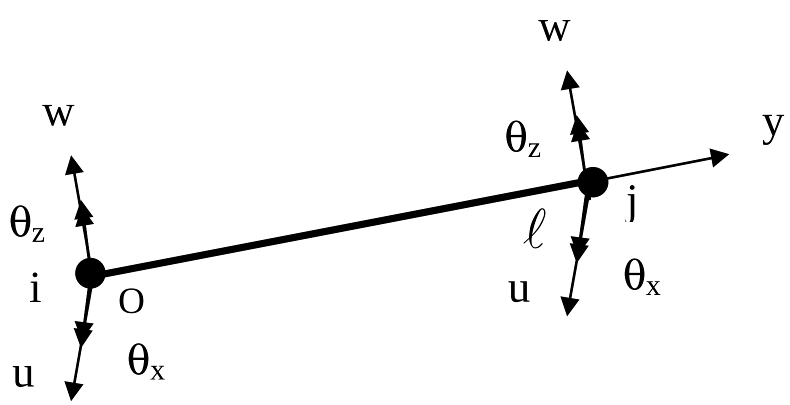


Figure 3. Beam FE $\langle \delta_n^e \rangle = \langle u_i, w_i, \theta_{xi}, \theta_{zi}, u_j, w_j, \theta_{xj}, \theta_{zj} \rangle$

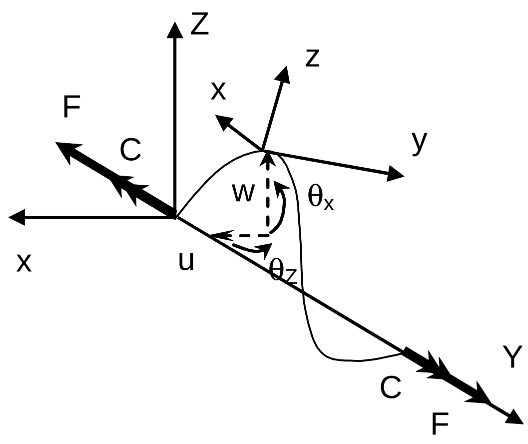


Figure 4

The assembly of the elementary matrices gives the following equations of the motion of the beam:

$$\left[M^{FE}\right]\{\ddot{\delta}\} + \left(\left[K^{FE}\right] + F\left[K_F^{FE}\right] + C\left[K_c^{FE}\right]\right)\{\delta\} = 0 \qquad (11)$$

where:

$\{\delta\}$ nodal displacement vector

$\left[M^{FE}\right]$, $\left[K^{FE}\right]$, symmetric mass and stiffness matrices.

$\left[K_F^{FE}\right]$, $\left[K_C^{FE}\right]$, stress stiffening matrices due to the force F (Nelson & McVaugh 1976) and to the torque C (Zorzi & Nelson 1980).

III- Theoretical results

1- Natural frequencies (Constant axial loads)

Numerical applications: a clamped-clamped rod Length: 0,494m, cross-section area: 5,35e-5m²,

$$\left[M^{FE}\right]\left\{\ddot{\delta}\right\}+\left(\left[K^{FE}\right]+F\left[K_F^{FE}\right]+C\left[K_c^{FE}\right]\right)\left\{\delta\right\}=0 \qquad (12)$$

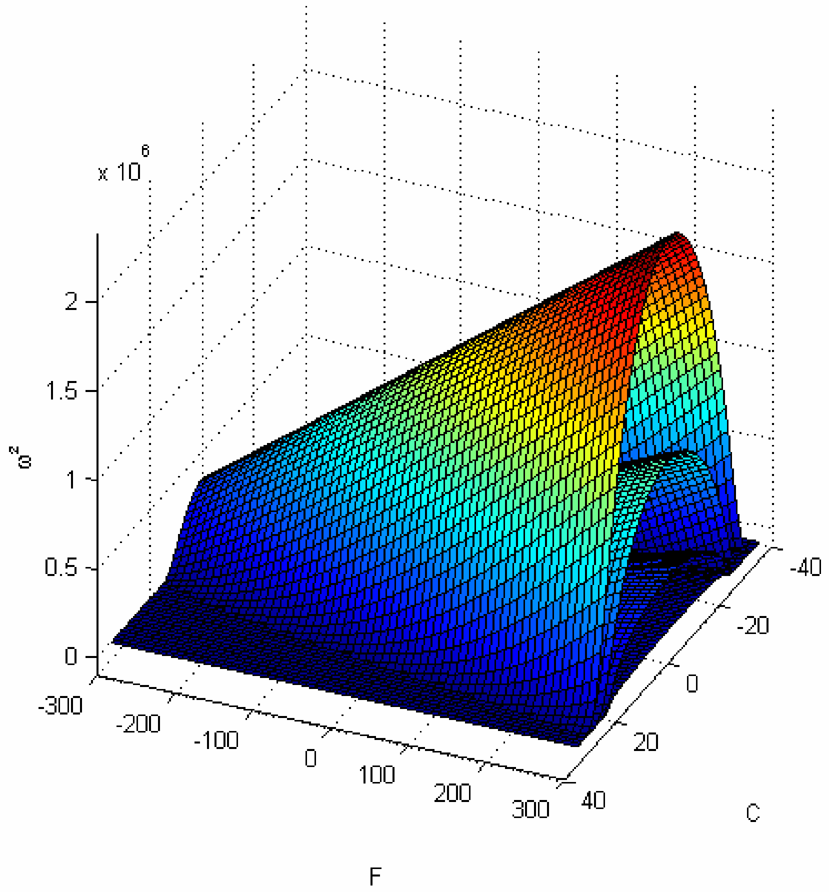


Figure 5

2- Dynamic stability analysis (Pulsating axial loads)

$$F = F_0 + F_1 \sin\eta_F t \qquad (13) \qquad\qquad C = C_0 + C_1 \sin\eta_C t \qquad (14)$$

State equations:

$$\{\dot{Y}(t)\} = [A(t)]\{Y(t)\} \qquad (15)$$

Monodromy matrix D: $\{Y(t+\tau)\} = [D]\{Y(t)\}$ (16)

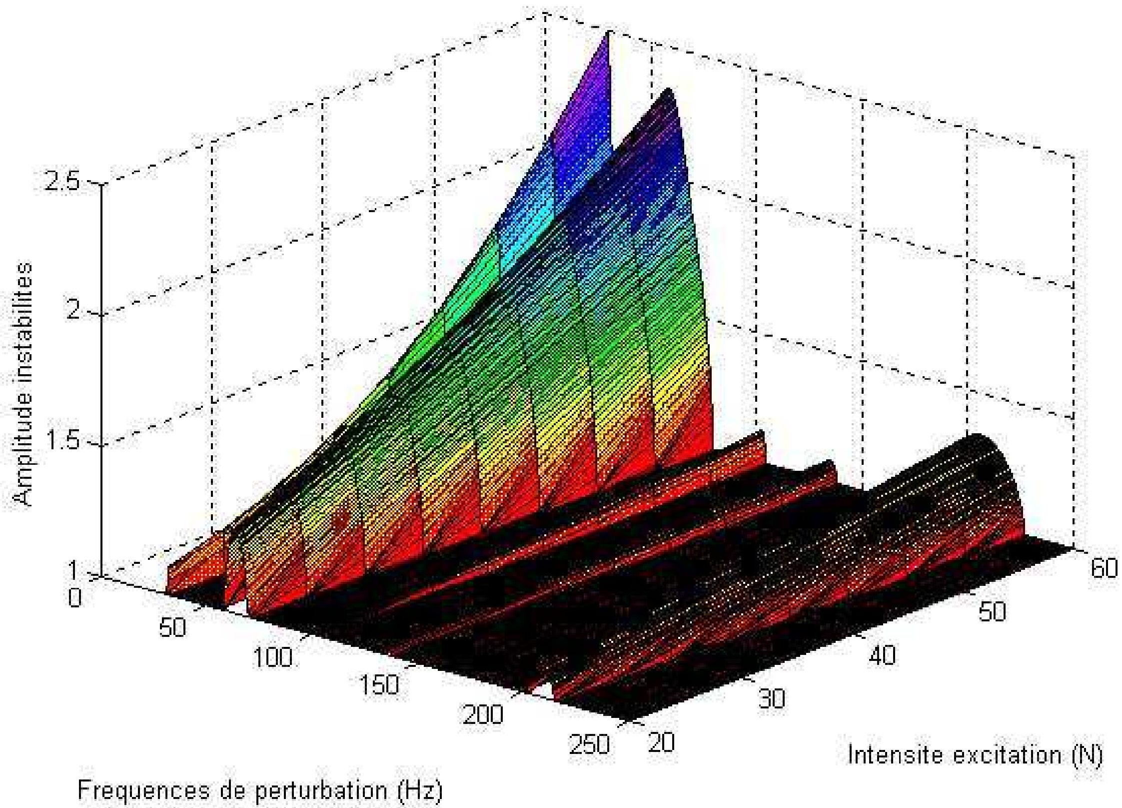
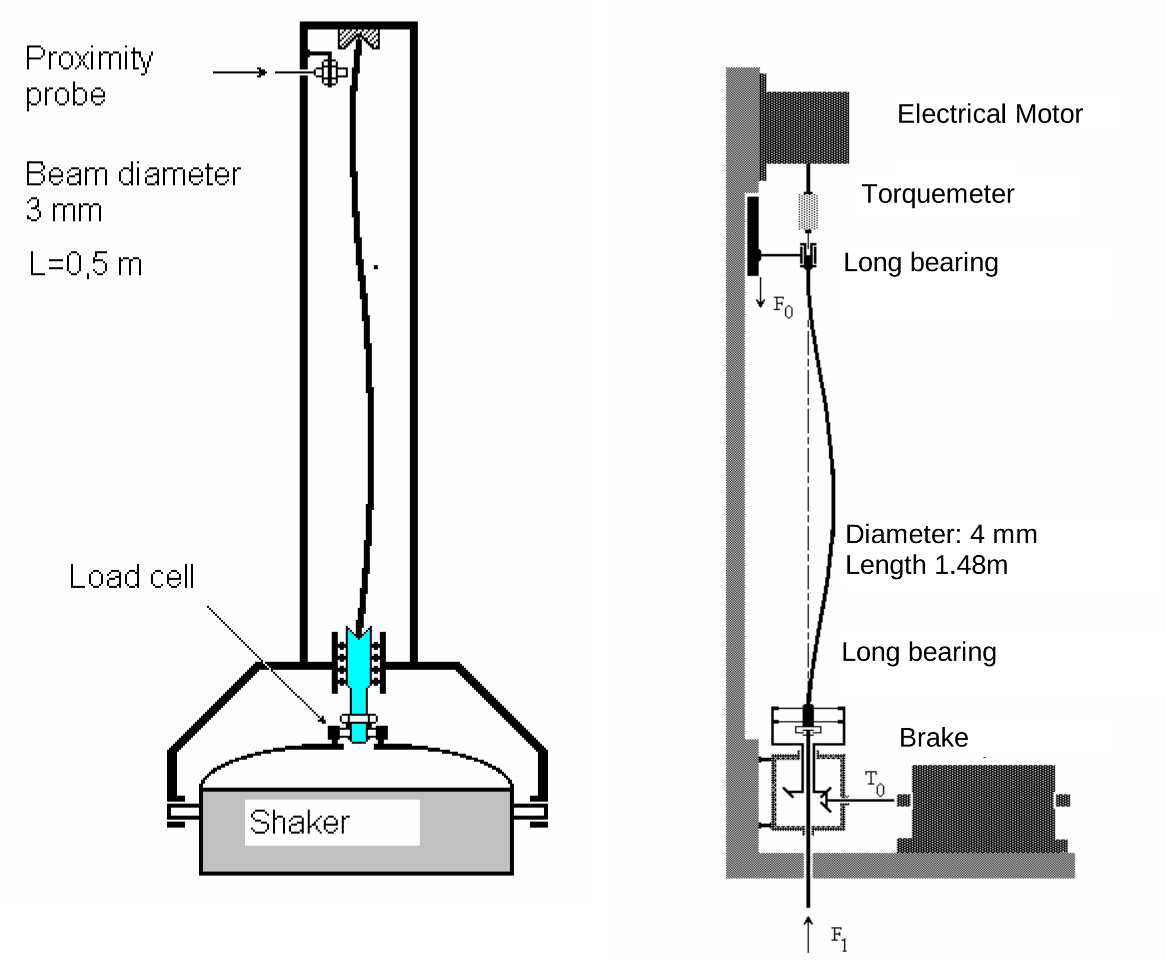


Figure 6. Lateral instability chart of the beam subject to an axial pulsating force.

V- Experimental investigation & results

Device #1

$F=F_0+F_1\sin\eta t$
Compression at rest

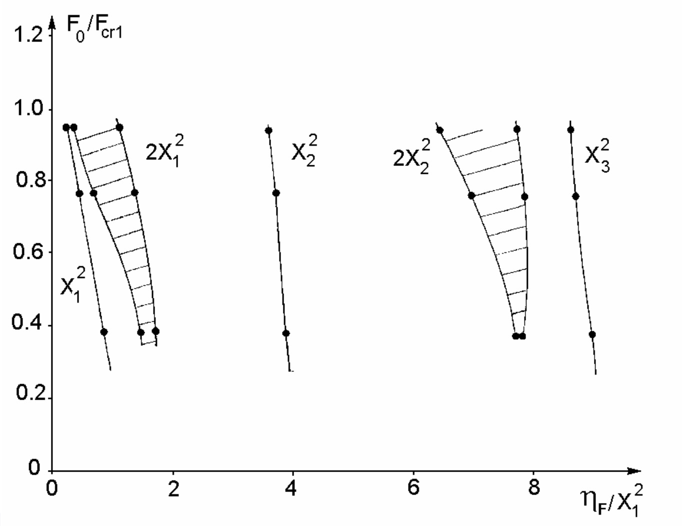


Device #2

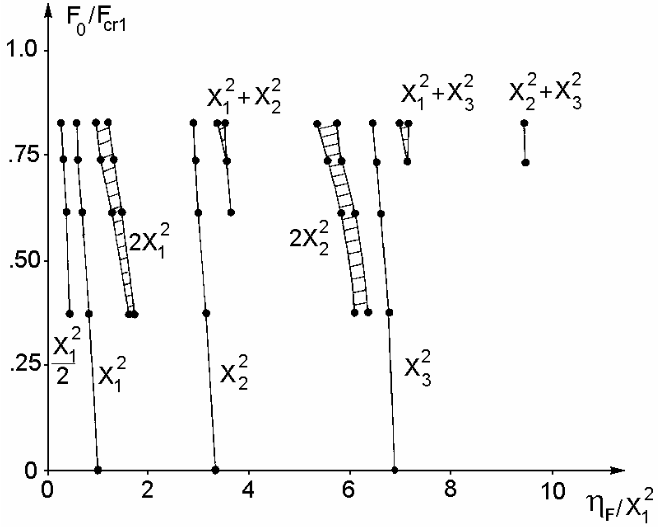
$F=F_0+F_1\sin\eta t$
$C=C_0+C_1\sin\eta t$
Compression, traction, torsion
At rest and in rotation
Clamped-Clamped beam

Figure 7. Experimental set-up

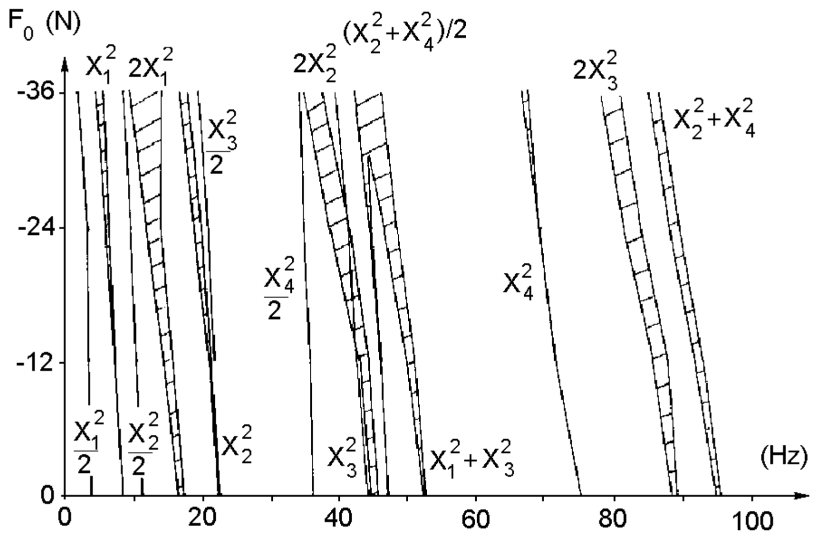
Experimental results from device #1. Beam subject to an axial pulsating force $F=F_0+F_1\sin\eta_F t$



SS-SS Beam



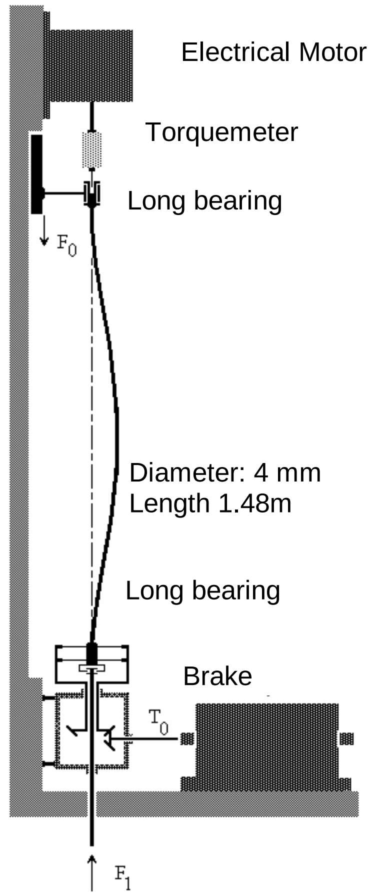
C-SS beam



C-C beam

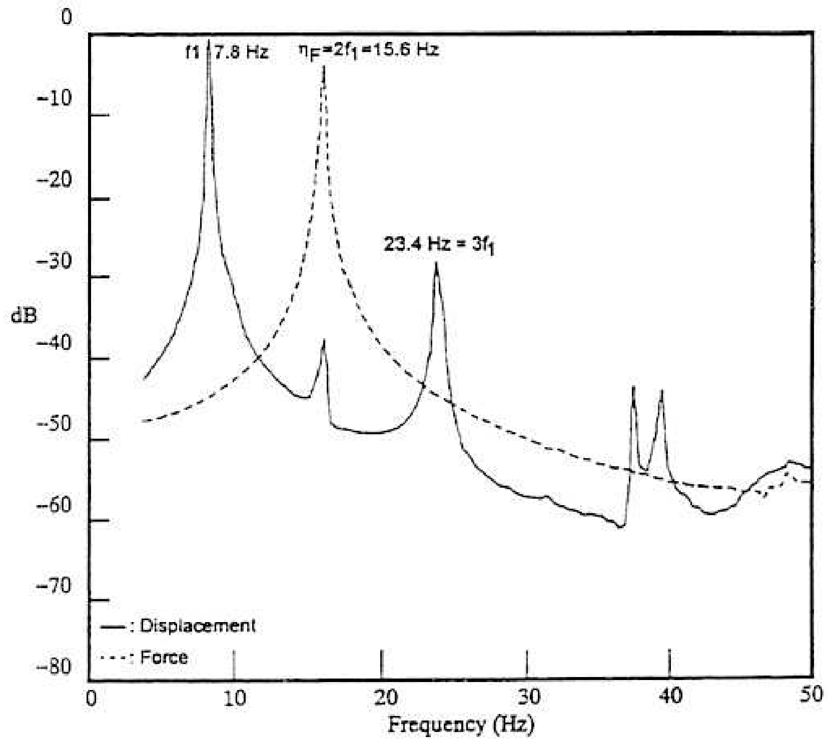
Figure 8. Measured instability regions

Experimental results from device #2. Clamped-Clamped Beam subject to $F=F_0+F_1\sin\eta_F t$



Hz	f_1	f_2	f_3	f_4
Mesure	7.8	19.8	36.6	48.4
Calcul	7.9	19.5	36.1	57.9

Natural frequencies



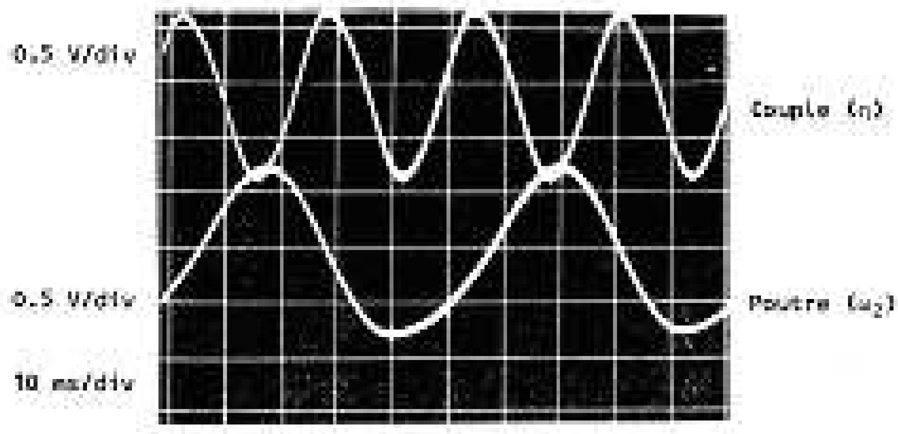


Figure 33 : Analyse temporelle du type d'instabilité 2 P (ici $\eta = 2\,\omega_2$)

Figure 9. Response: instability $\eta_F \sim 2f_1$

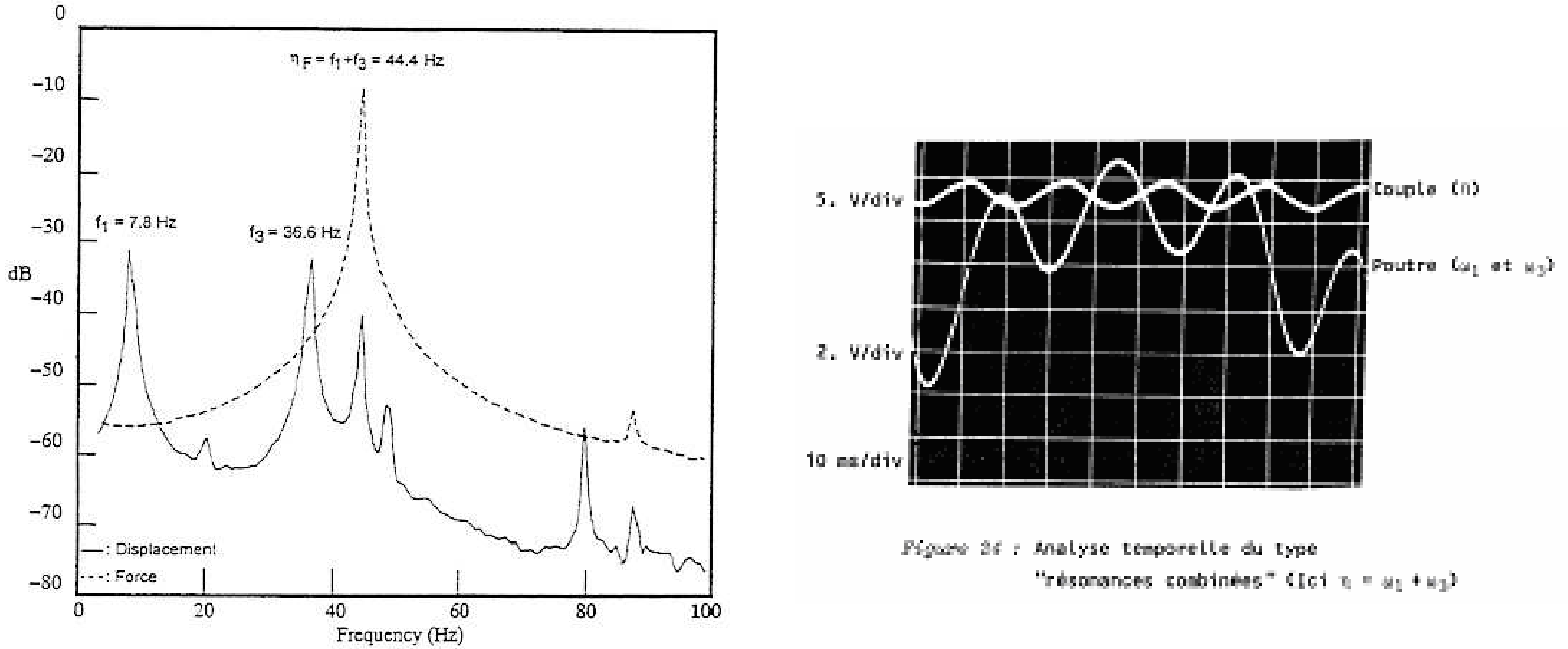


Figure 10. Response: instability $\eta_F \sim f_1+f_3$

Predicted lateral response of the clamped-clamped beam subject to $F=F_0+F_1 \sin\eta_F t$

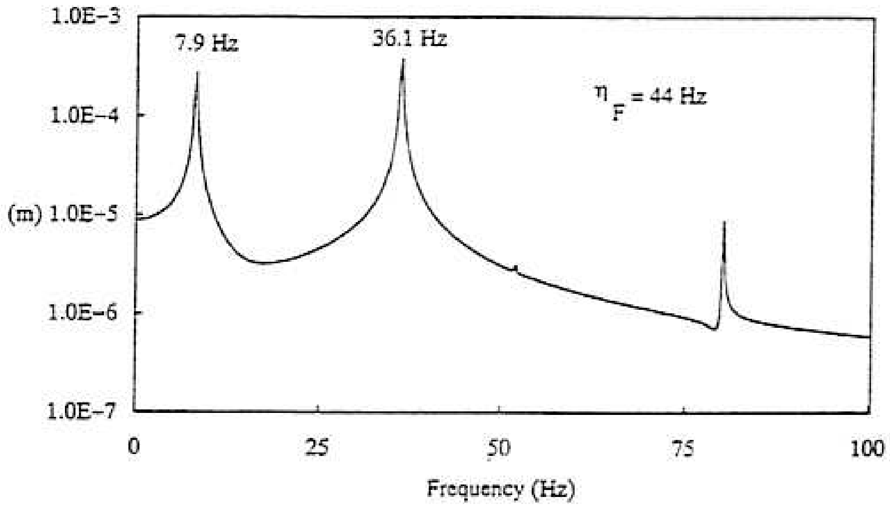


Figure 12. Response in the frequency domain $\eta_F \sim f_1+f_3$

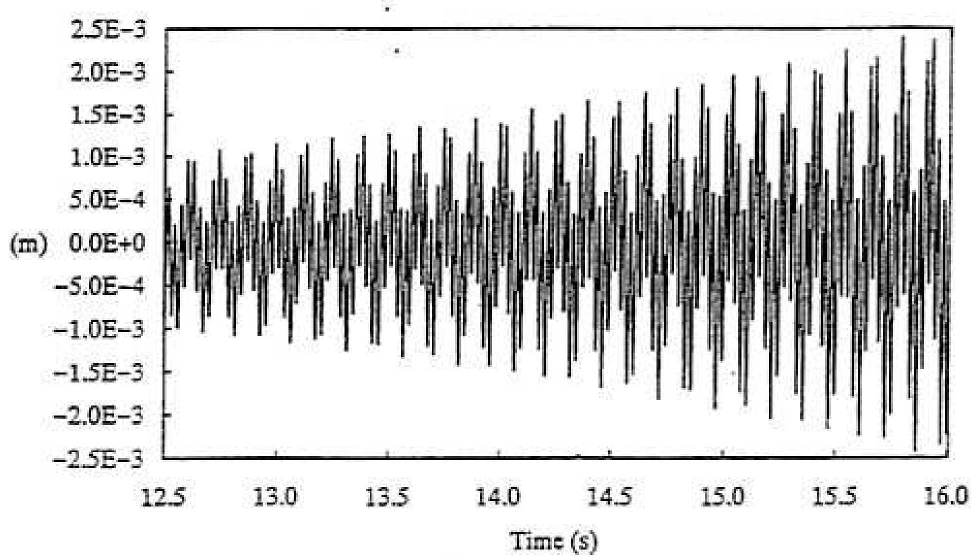


Figure 11. Response in the time domain $\eta_F \sim f_1+f_3$

VI- Conclusions

- Linear system with time-varying parameters exhibit instabilities
- Instability charts useful for designing structures and machines
- The instability region are reduced in the presence of damping
- Instability regions depends on
  - o the type of parametric excitation (force or torque)
  - o and the type of boundary conditions
- Lateral instabilities when the forcing frequency $\eta \approx \frac{\omega_i \pm \omega_j}{k}$,
  - o ω_i, i^{th} natural angular frequency,
  - o k, instability order: primary instability (k=1), secondary instability, (k=2), etc.

$\eta \approx \frac{\omega_i \pm \omega_j}{k}$	**Simply supported Simply supported**	**Clamped Simply-supported**	**Clamped-clamped**
$\mathbf{F_0+F_1 \sin \eta t}$	i=j, $\forall$ k	$\forall$ i+j, k	i+j, l pairs,
$\mathbf{C_1 \sin \eta t}$	i+j, k odd i+j, k even	i+j, k odd i+j, k even j-i even, k odd	i+j, l odd i+j, l even
$\mathbf{C_0+C_1 \sin \eta t}$	$\forall$ i+j, k	unstable	$\forall$ i+j, k

Dynamic instability synthesis