# Reliability Based Optimisation of Aeronautical Composite Structures Under Aeroelastic Constraints


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# Presentation
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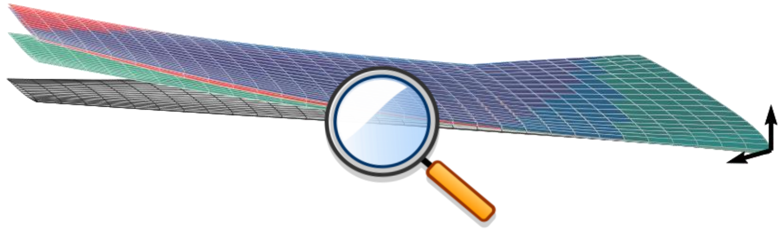
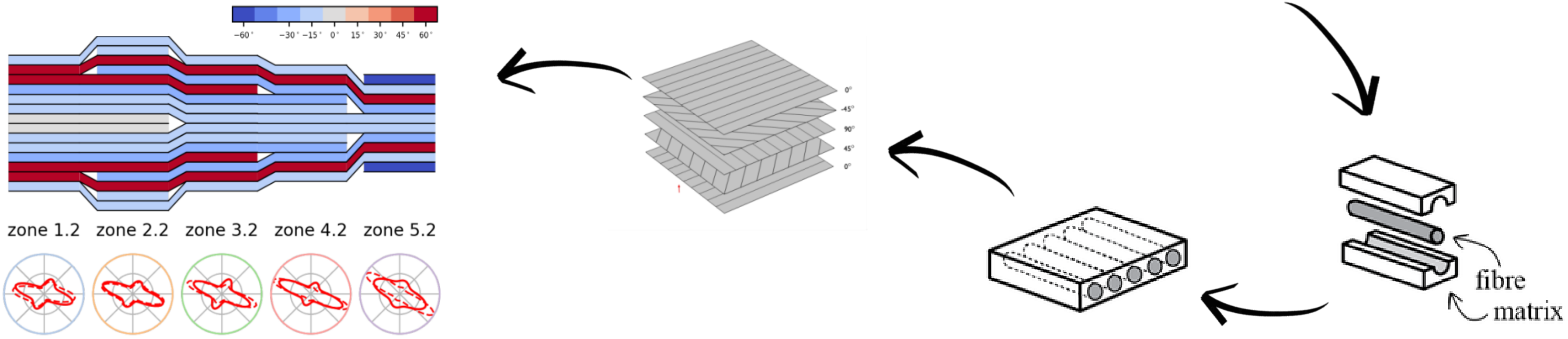
# Problem introduction

## *Multi-scale design space for composite materials*

**Composite structures:**

- Series of fibres (carbon, glass, Kevlar, etc.) being held together by a matrix (epoxy, polyester, PP, etc.)
- Disposed into fine unidirectional layers → plies
- N number of plies are stacked to form a laminate with a defined stacking sequence $(\theta_1, \ldots, \theta_N)$
- Different zones can be defined within the same problem

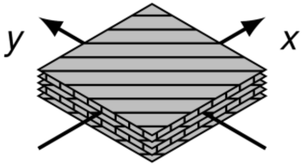


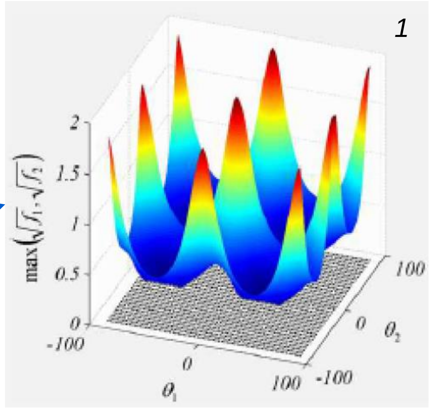


# Problem introduction
## *Multi-scale design space for composite materials*

**Laminate stacking sequences:**

- Large number of degrees of freedom:
  $A(\theta_1, \dots, \theta_N)$, $\mathrm{B}(\theta_1, \dots, \theta_N)$ and $\mathrm{D}(\theta_1, \dots, \theta_N)$
- Not a constant number of degrees of freedom
  $$\min y = f(\theta_1, \dots, \theta_N)$$
- Usually, many local minimal values can be found in optimisation problems



# Problem introduction

## *Multi-scale design space for composite materials*

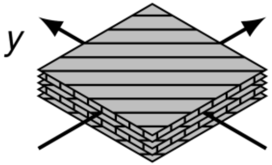
### Laminate stacking sequences:

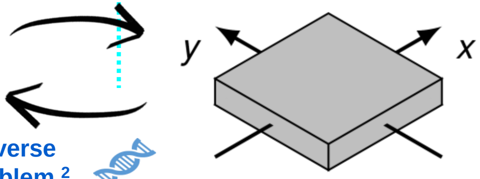
- Large number of degrees of freedom:
  $A(\theta_1, \dots, \theta_N)$, $\mathrm{B}(\theta_1, \dots, \theta_N)$ and $\mathrm{D}(\theta_1, \dots, \theta_N)$
- Not a constant number of degrees of freedom
  $$\min y = f(\theta_1, \dots, \theta_N)$$
- Usually, many local minimal values can be found in optimisation problems

### Lamination Parameters (material homogenisation):

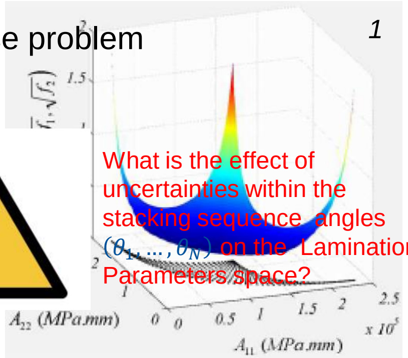
- Constant number of variables at each zone (13):
  $A(v_1^A, v_2^A, v_3^A, v_4^A)$, $\mathrm{B}(v_1^B, v_2^B, v_3^B, v_4^B)$ and $D(v_1^D, v_2^D, v_3^D, v_4^D)$
  $$\min y = f(v, N)$$
- "Convex/smooth" optimisation space
- Compatibility conditions
- No exact solution to the inverse problem

$$\mathbf{v}^{\mathbf{A}}_{[1,2,3,4]} = \frac{1}{t}\int_{-h/2}^{h/2} [\cos(2\theta_k), \sin(2\theta_k), \cos(4\theta_k), \sin(4\theta_k)]\, dz$$
$$\mathbf{v}^{\mathbf{B}}_{[1,2,3,4]} = \frac{4}{t^2}\int_{-h/2}^{h/2} [\cos(2\theta_k), \sin(2\theta_k), \cos(4\theta_k), \sin(4\theta_k)]\, z dz$$
$$\mathbf{v}^{\mathbf{D}}_{[1,2,3,4]} = \frac{12}{t^3}\int_{-h/2}^{h/2} [\cos(2\theta_k), \sin(2\theta_k), \cos(4\theta_k), \sin(4\theta_k)]\, z^2 dz$$
1




Inverse Problem [2]

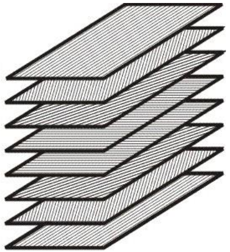



What is the effect of uncertainties within the stacking sequence angles $(\theta_1, \dots, \theta_N)$ on the Lamination Parameters space?

# Problem introduction

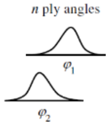
*Multi-scale design space for composite materials*

## Laminate stacking sequences:



$(\theta_1, \ldots, \theta_N)$

**Inverse Problem**

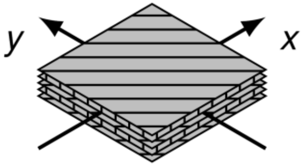



$\Theta\ defined\ by\ N(\mu_\Theta, \sigma_\Theta)$

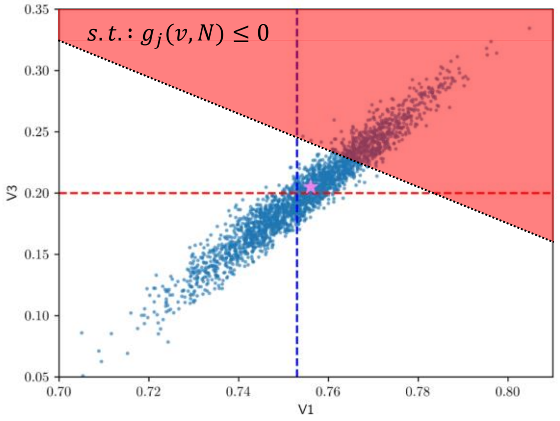
$$\mathbf{v}^{\mathbf{A}}_{[1,2,3,4]} = \frac{1}{t}\int_{-h/2}^{h/2} [\cos(2\theta_k), \sin(2\theta_k), \cos(4\theta_k), \sin(4\theta_k)]\, dz$$

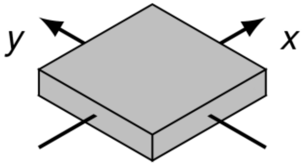
$$\mathbf{v}^{\mathbf{B}}_{[1,2,3,4]} = \frac{4}{t^2}\int_{-h/2}^{h/2} [\cos(2\theta_k), \sin(2\theta_k), \cos(4\theta_k), \sin(4\theta_k)]\, z dz$$

$$\mathbf{v}^{\mathbf{D}}_{[1,2,3,4]} = \frac{12}{t^3}\int_{-h/2}^{h/2} [\cos(2\theta_k), \sin(2\theta_k), \cos(4\theta_k), \sin(4\theta_k)]\, z^2 dz$$




## Lamination Parameters (material homogenisation):




Importance of studying **failure probability** if limit state functions exist

$\mathbb{P}(g_j(v, N) < 0) \leq \mathbb{P}^{max}$

# Problem introduction
## *Methodology development*

[1] *Jiao et el. (2018) Genetic and Evolutionary Computation.*

- Aeroelastic models can be computationally expensive
- Possible big number of design variables doesn't allow for an overall design space computation
- Constrained Optimisation Process (COP) [1] →

$$\min f(v,N)$$
$$s.t.: g_j(v,N) \leq 0$$
$$s.t. Compatibility\ Conditions$$

- Error in fiber direction ($\Theta$) while stacking composite plies introduces a reliability based analysis on the constraints:

$$\min f(v(\mu_\Theta),N)$$
$$s.t.: \mathbb{P}(g_j(v(\Theta),N) < 0) \leq \mathbb{P}^{max}$$
$$s.t.\ Compatibility\ Conditions$$

- Need for an inverse problem solver that can address the reliability based design

## Multi Step Optimisation Strategy:

1. Global design space exploration via Efficient Global Optimisation (EGO) with deterministic approach
2. Genetic Algorithm (GA) optimisation with Reliability Based Design Optimisation (RBDO) approach for inverse problem

# Multi Step-Optimisation Strategy

## *Workflow proposed*

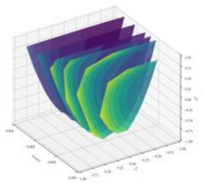
**Multi Step Optimisation Strategy:**

1. Global design space exploration via Efficient Global Optimisation (EGO) with deterministic approach
2. Genetic Algorithm (GA) optimisation with Reliability Based Design Optimisation (RBDO) approach for inverse problem

### 1. Efficient Global Optimisation (EGO)[1]

*Initial coarse DoE*

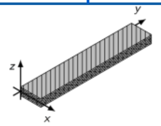
**Surrogate-KRG**



**Acquisition function optimisation**

*COP convergence check*

**Next Refinement Point**

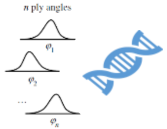


**NASTRAN**

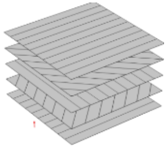
### 2. Reliability Based Design Optimisation (RBDO)

**Validated Surrogate model for RBDO optimisation**

**Genetic Algorithm**

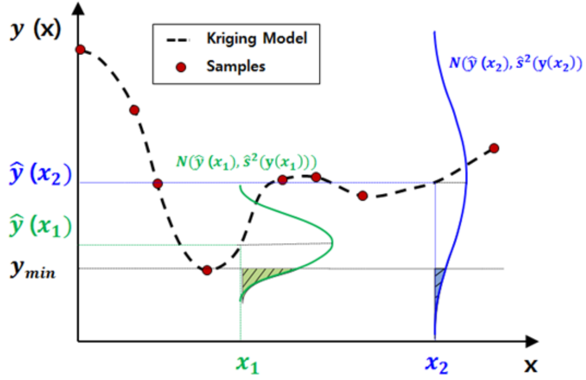


***Final Stacking Sequence*** $(\theta_1, \dots, \theta_N)$



# Efficient Global Optimisation
## *Acquisition Functions*

$$\begin{cases} X = \{x_1, x_2, \dots, x_n\} \\ Y = \{y_1, y_2, \dots, y_n\} \end{cases} \rightarrow f_{min} = min\{y_1, y_2, \dots, y_n\}$$




- Expected Improvement[1] (EI):

$$\mathbb{E}[I(x)] = \mathbb{E}[(f_{min} - Y(x), 0)]$$

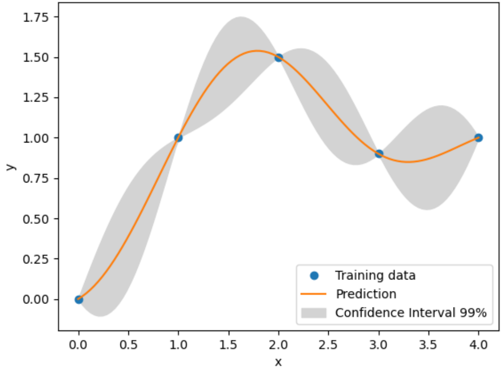
→ Depends on the surrogate model of the objective function

- Probability of Feasibility (PoF):

$$PoF = \mathbb{P}(g_{const.}(x) < 0)$$

→ Depends on the surrogate models of the constraints

$$\boxed{\boldsymbol{objective} = PoF * \mathbb{E}[I(x)]}$$



# Reliability Based Design Optimisation

*Methodology development*

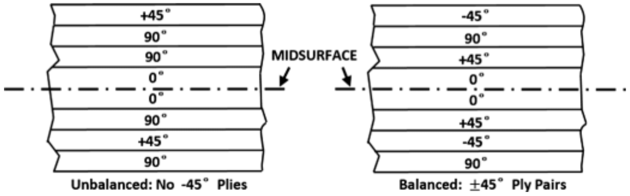
- Composite tailoring problem using the identified surrogate models $(\tilde{f}, \widetilde{g_j})$: finding a stacking sequence $(\theta_1, \dots, \theta_N)$ :

$$\min \tilde{f}(v(\mu_\Theta), N)$$
$$s.t.: \mathbb{P}\left(\widetilde{g_j}(v(\Theta), N) < 0\right) \leq \mathbb{P}^{max}$$
$$s.t.\ Design\ Rules$$

- Solved via Genetic Algorithm (GA) given the numerous local minima within the solution space and non-continuous design space

- Monte Carlo analysis used to evaluate the probability of violating a constraint

- Design rules can be imposed by the manufacturer and must be taken into consideration within the inverse problem
  - Symmetry
  - Balanced composite
  - Given orientation for the outer plies
  - etc.



# Application case: Aeroelastic Composite Plate [1]

## *Problem presentation*

- Analysis via NASTRAN (DLM)[1]

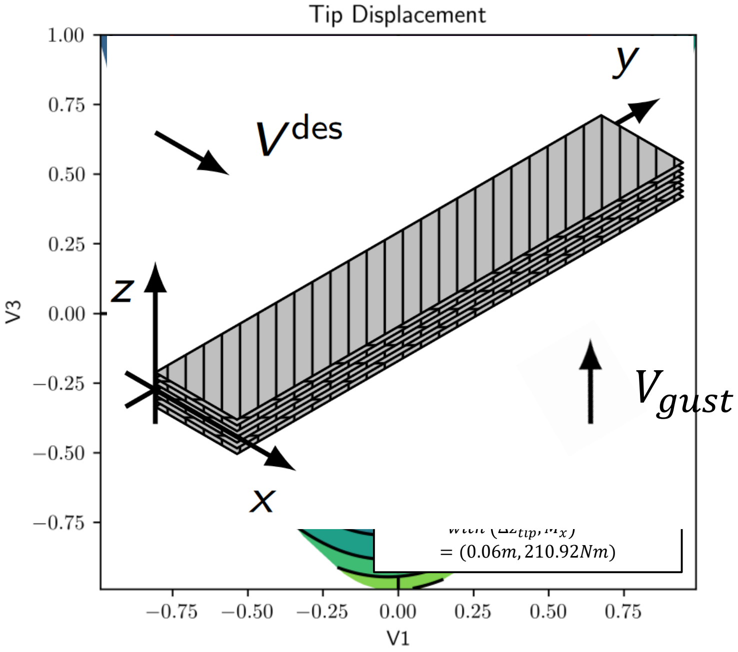
- Two design variables $(V_D^1, V_D^3)$

- **Constraint:** Fixed fluid velocity ($V_{des}$) and lift force ($L$)
  - $V_{des} = 105\ {}^{m}/_{s}$
  - Constant lift $L = {}^{1}/_{2}\,\rho V_{des}^2 S C_l$
  - Free AoA ($\alpha$)
  - → ***Output***: Tip displacement (mm)

$$\mathbb{P}(\ g_{disp.}(V_D^1, V_D^3) < 0) \leq \mathbb{P}^{max}$$

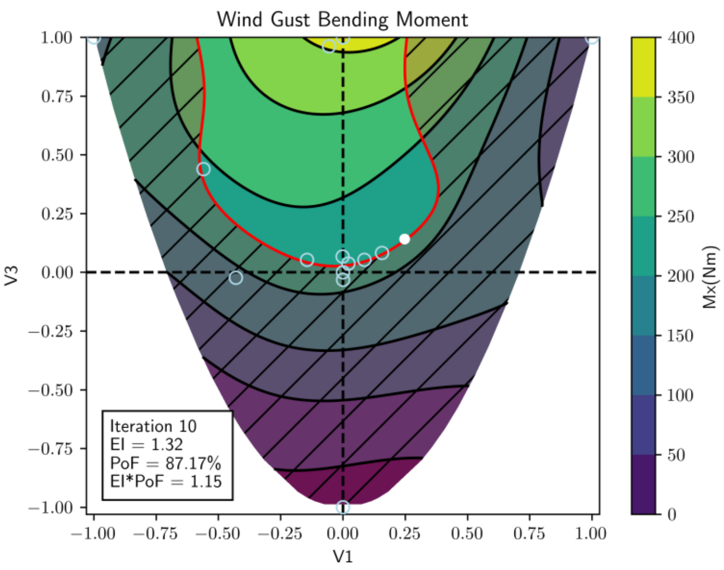
- **Objective:** Fixed fluid velocity ($V_{des}$) + vertical gust ($V_{gust}$)
  - $V_{gust} = 1\% * V_{des} = \ 1{,}05\ {}^{m}/_{s}$
  - Sinusoidal waveform with a frequency of 20Hz
  - → ***Output***: Maximum bending moment at base (Nm)

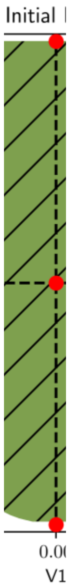
$$\min(M_x(V_D^1, V_D^3))$$

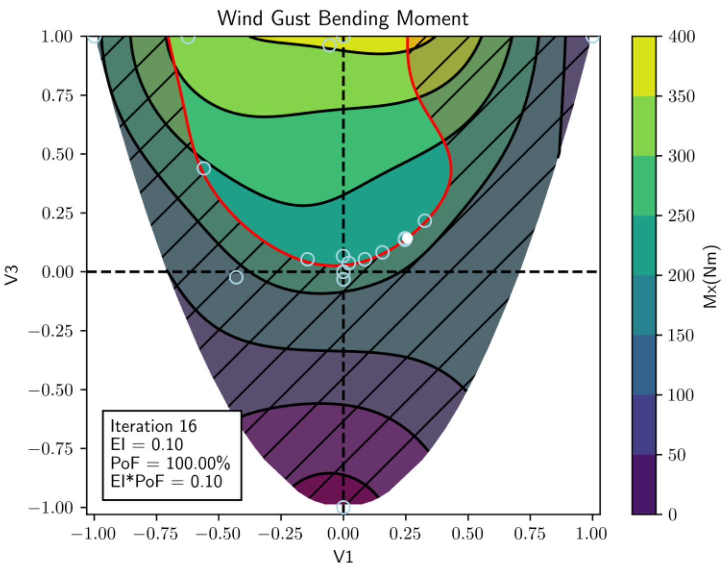


# Application case: Aeroelastic Composite Plate

## *Results – Step 1: EGO exploration*



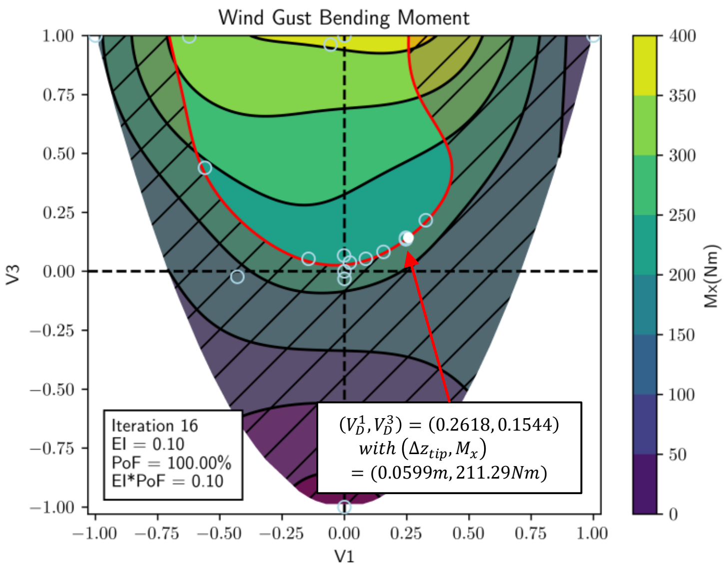



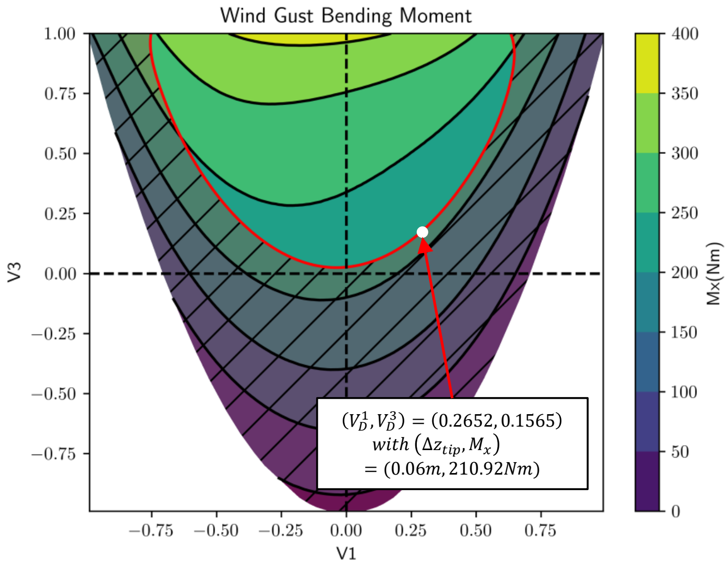
# Application case: Aeroelastic Composite Plate

## *Results – Step 1: EGO exploration*

**Identified Surrogate Model**

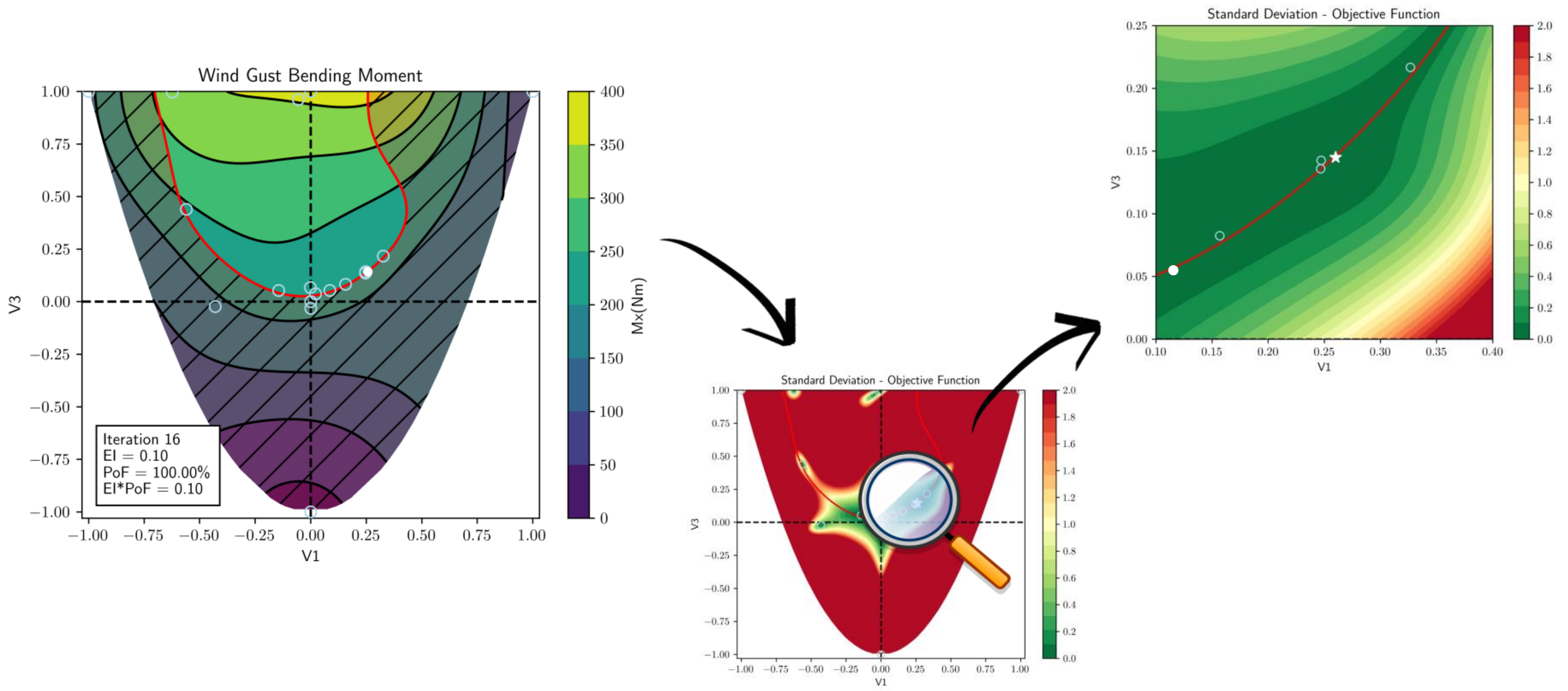



**Reference Model**



# Application case: Aeroelastic Composite Plate

## *Results – Final surrogate model vs. Real response*



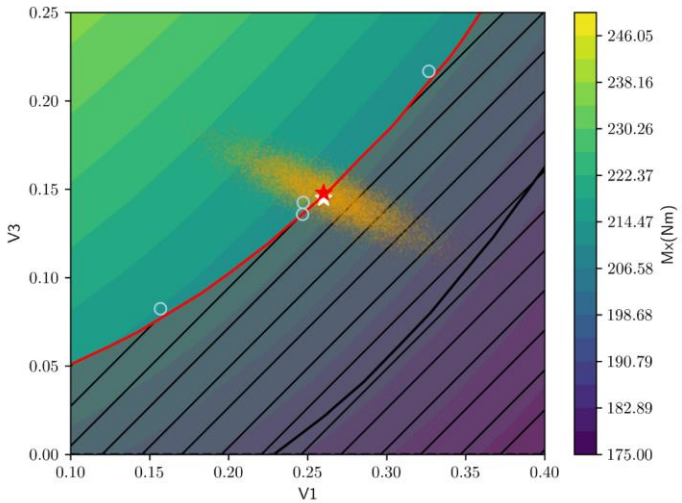
[1] Irisarri et al. (2009) Composite Science and Technology.

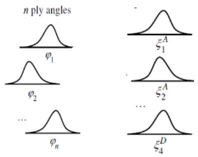
# Application case: Aeroelastic Composite Plate

## *RBDO – Results*

**Deterministic Solution**
*+ Classic Composite Inverse Problem* [1]

- $(V1, V3) = (0.262, 0.145)$
- $M_x = 211.32\ Nm$
- $\mathbb{P}\left(\widetilde{g_j}(v(\Theta), N) < 0\right) = 0{,}53$

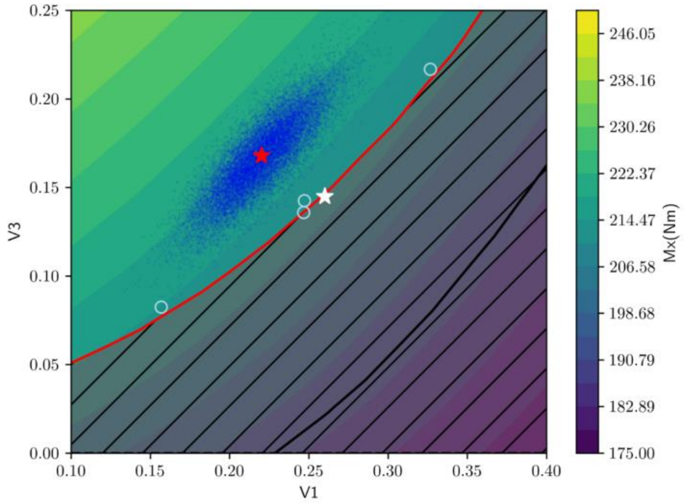



**R.B.D.O**

$$\min \tilde{f}(v(\mu_\Theta), N)$$
$$s.t.: \mathbb{P}\left(\widetilde{g_j}(v(\Theta), N) < 0\right) \leq \mathbb{P}^{max}$$

- $(V1, V3) = (0.221, 0.168)$
- $M_x = 218.52\ Nm$
- $\mathbb{P}\left(\widetilde{g_j}(v(\Theta), N) < 0\right) = 0{,}001$



# Application case: Aeroelastic Composite Plate
## *Conclusions*

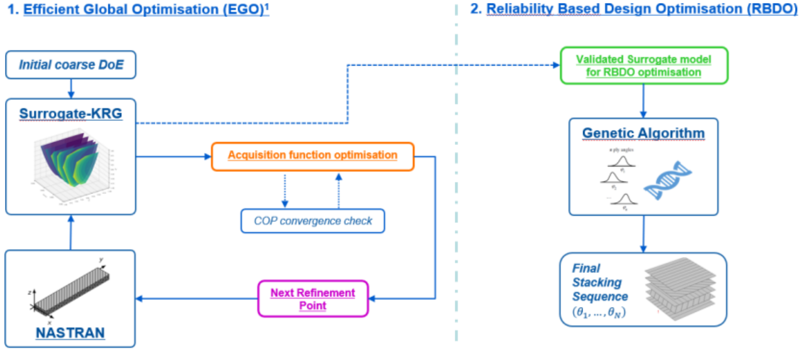
**EGO:**

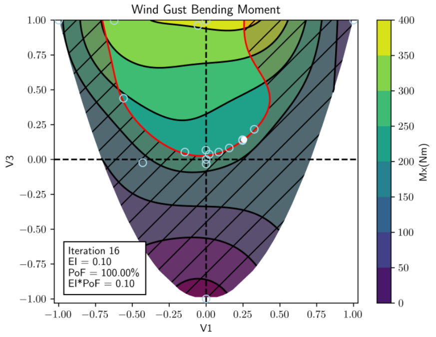
- ➢ Active exploration (deterministic) of the design space
- ➢ Efficient construction of a surrogate model for RBDO

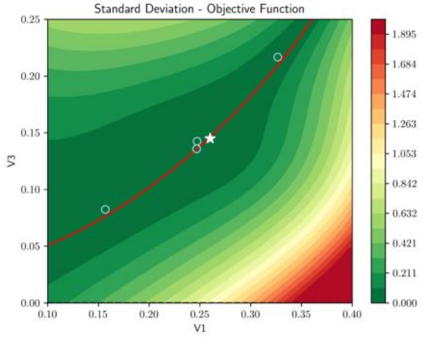
**RBDO:**

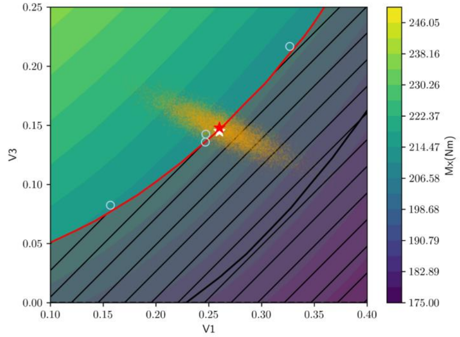
- ➢ Optimal solution respecting failure probability
- ➢ The surrogate models permits the use of Monte Carlo analysis

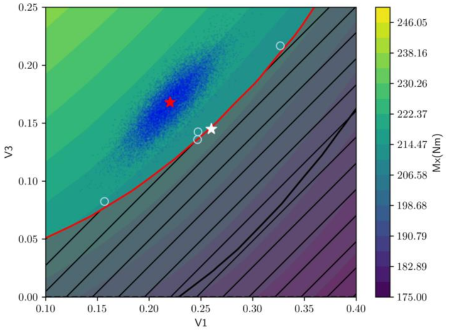
→ Efficient Multi-Step RBDO process



*Experimental images courtesy of A.Lepage and F.Nicolas (DAAA, ONERA)*
*[1] Fabbiane et al. (2022) CEAS AeroNaut. J.*

# Next steps
## *Towards "real" aeronautical studies*

**Next step: Multi-zone + ply orientation freedom**

- Augment the number of design variables in the plate design

$$(v_1^D, v_2^D, v_3^D, v_4^D, N)_p$$

**Further steps: CARACAL** [1]

- *Conception aéroélastique pour la réduction de charge à la rafale*
- Free fiber direction + multi-zone design (10 zones total)
  → Up to 40 design variables
- Multi-fidelity aerodynamic problem simulation

**Aerodynamic Solver:**

***Low-Fidelity(DLM)***

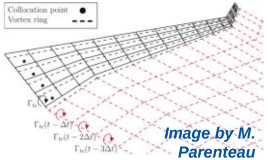


*Image by M. Parenteau*

***High-Fidelity(CFD)***

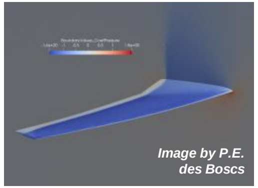
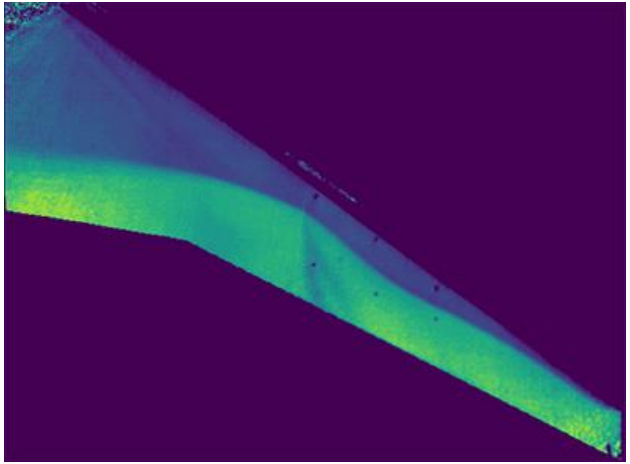
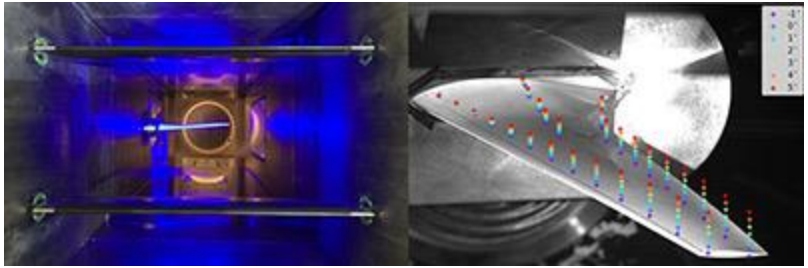


*Image by P.E. des Boscs*





# Thank you for your attention

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# ANNEX:

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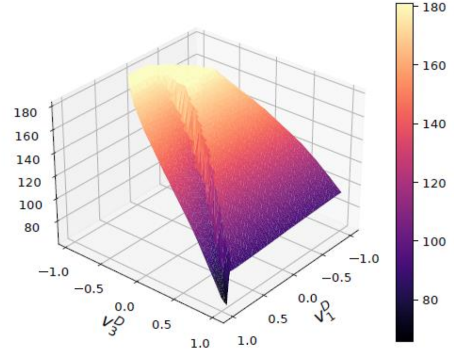
# Context and objectives

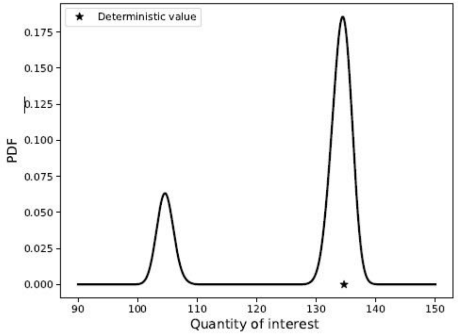
## *Bibliographical work*

- Reliability-based design optimization of composite laminates for aeroelastic applications (Coelho 2023; Sharifi 2023)

- Ply orientations uncertainty has an influence on some instabilities:
  - Buckling (Conceicao et al, 2017; Wang et al., 2017; Pagani and Sanchez, 2022)
  - Flutter velocity (Scarth et al., 2014, Nitschke et al. 2019)

- Efficient global optimisation:
  - EGO (Jones, D. R., Schonlau, M., & Welch, W. J. 1998)
  - Expected Improvement (Ginsbourger et al. 2010)
  - Constrained EI (Jiao et al. 2018)



# Inverse problem
## *Design space*

- **Simplification of the LP problems to a 2 dimension space:**
  - Symmetrical laminate condition $(\theta_1, \theta_2, \dots, \theta_n, \theta_n, \dots, \theta_2, \theta_1)$
  - Study cases → Pure bending/flexural cases
  - Balanced laminate condition: $\forall \theta_N \ \rightarrow \exists - \theta_N$
  - ➢ Only $v_1^D$ and $v_3^D$ are left after the simplifications
  - ➢ Angles $\pm[0°, 15°, 30°, 45°, 60°, 75°, 90°]$

$$\cancel{(v_1^A, v_2^A, v_3^A, v_4^A)}$$
$$\cancel{(v_1^B, v_2^B, v_3^B, v_4^B)}$$
$$(v_1^D, \cancel{v_2^D}, v_3^D, \cancel{v_4^D})$$

- **$v_{[1,3]}^D$ Definition:**

$$v_{[1,3]}^D = \frac{12}{t^3} \int_{-h/z}^{h/2} [cos(2\theta_k), cos(4\theta_k)] z^2 \, dz$$

$$v_{[1,3]}^D = \frac{12}{t^3} \sum_{\varkappa}^{N} \frac{z_k^3 - z_{k-1}^3}{3} [cos(2\theta_k), cos(4\theta_k)]$$

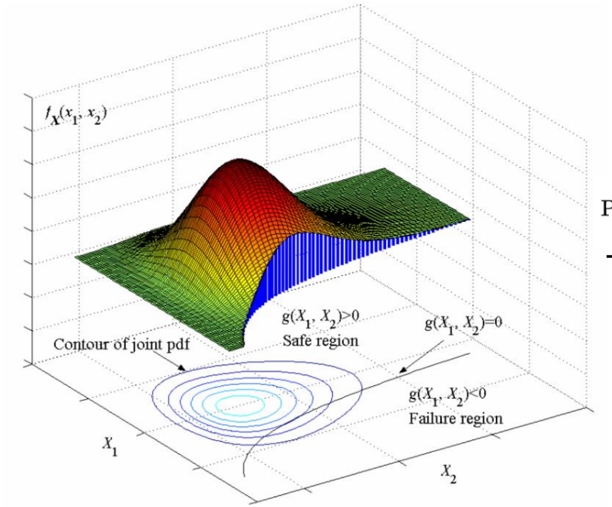
$$R_r = (0^0, \dots, 45^0)$$
$$R_l = (45^0, \dots, 90^0)$$

- **Design space limits:**

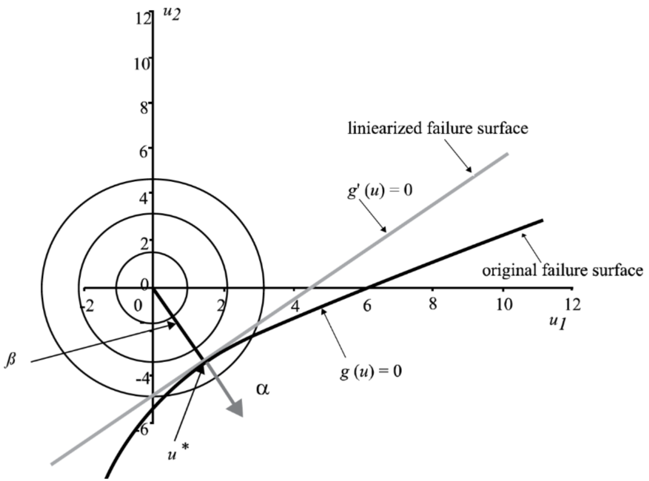
$$v_3^D \leq (v_1^D)^2 - 1$$
$$-1 \leq v_{[1,3]}^D \leq 1$$

# Inverse problem
## *Failure probability analysis*

- **Proposed failure probability analysis methodology: First Order Reliability Method (FORM)**
  - The entire calculation space (Gaussian distributions) is normalised
  - The limit state function is linearised on the nearest point to the mean values




$$\mathrm{P_f} = \int_{\mathrm{F}} f_1(x_1) f_2(x_2) \cdots f_n(x_n) \mathrm{d}x_1 \mathrm{d}x_1 \cdots \mathrm{d}x_n$$

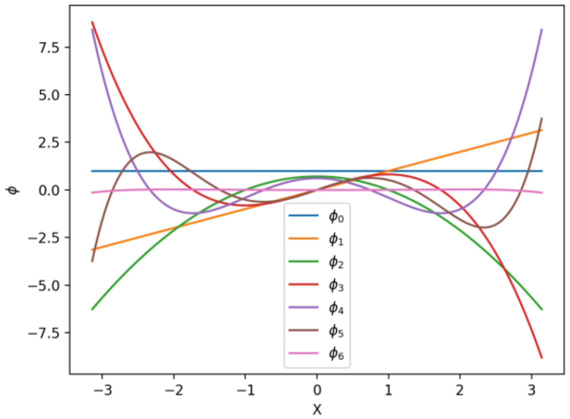


# Inverse problem

*Failure probability analysis*

- **Proposed failure probability analysis methodology: First Order Reliability Method (FORM)**
  - The entire calculation space (Gaussian distributions) is normalised
  - The limit state function is linearised on the nearest point to the mean values

- Quick and easy calculation
- Best approximation when failure probability is low
- Might underestimate failure probability if function is strongly convex




**We need $\mu_{v_1^D}$, $\mu_{v_3^D}$, $\sigma_{v_1^D}$, $\sigma_{v_3^D}$ and $corr(v_1^D, v_3^D)$ values to launch the algorithm**

- **Extraction of LPs distributions from reduced Monte Carlo analysis:**
  - Easy computation but high cost
- **Fourier Chaos Expansion (FCE):**
  - Method to compute the expectation of trigonometric functions

# Precedent studies – L.Coelho
*Study cases and results*

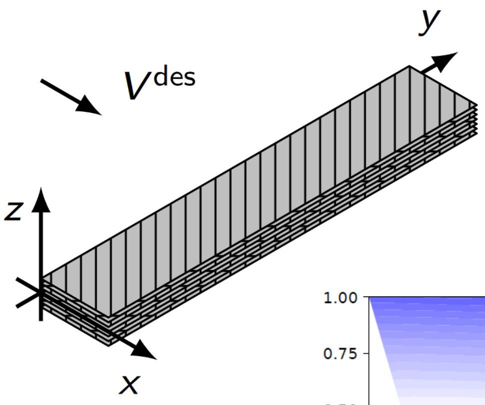
## Composite plate immersed in an airflow

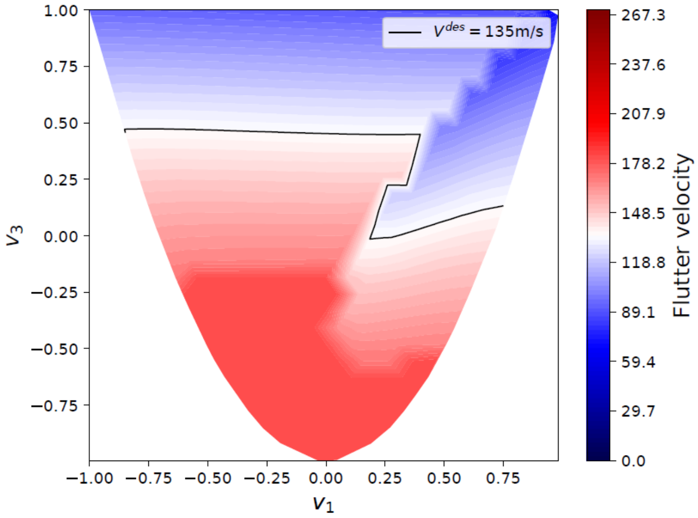
- Fixed fluid velocity ($V^{des}$) and angle of attack ($\alpha$):
  - $V^{des} = 135\,{}^{m}/_{s}$
  - $\alpha = 5°$
  - Two design variables $(V_D^1, V_D^3)$

- Optimize tip displacement + Respect flutter constraint (probabilistic description):

$$\min(-d_{tip}(V_D^1, V_D^3))$$
$$\mathbb{P}(g(V_D^1, V_D^3) < 0) \leq \mathbb{P}^{max}$$

- Surrogate model strategy:
  - Definition of a DoE
  - Solution on NASTRAN and computation of a kriging metamodel

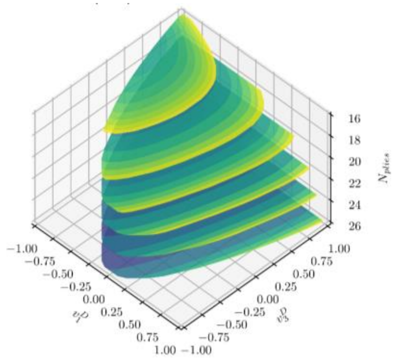


# Precedent studies – L.Coelho
## *Limitations and perspectives*

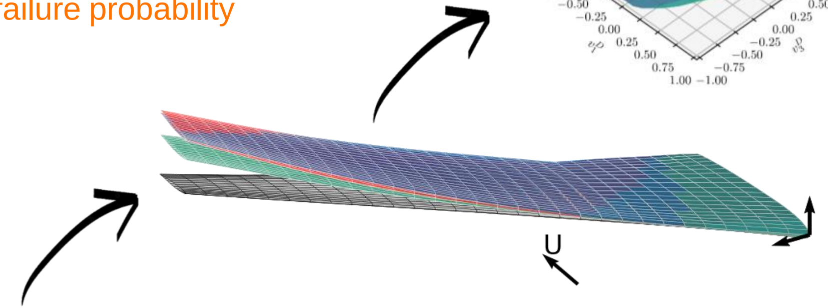
**Limitations:**

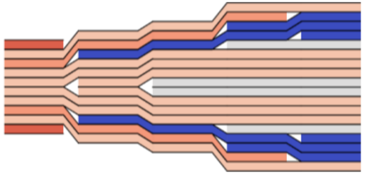
- Limited design space (no thickness)
- Convergence issues due to:
  - Inverse problem discrepancy from the expected LPs
  - No sensitivity on the failure probability when optimizer travels into failure domain
- Error introduced by the surrogate-model bigger than expected failure probability




**Perspectives:**

- Increase number of design variables:
  - Introduction of composite thickness
  - Introduction of multiple design zones
  - Towards more complex aeronautical problems
- Update surrogate modeling strategy
  - Adaptative kriging strategies



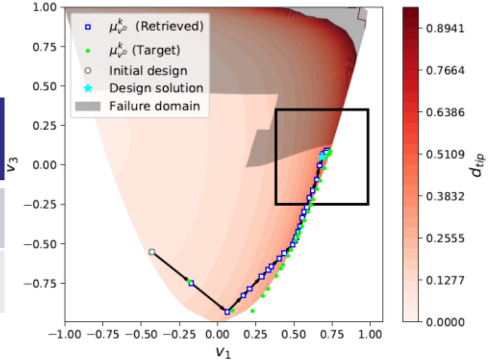



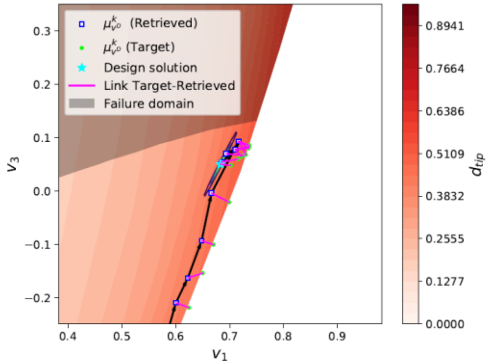
# Precedent studies – L.Coelho

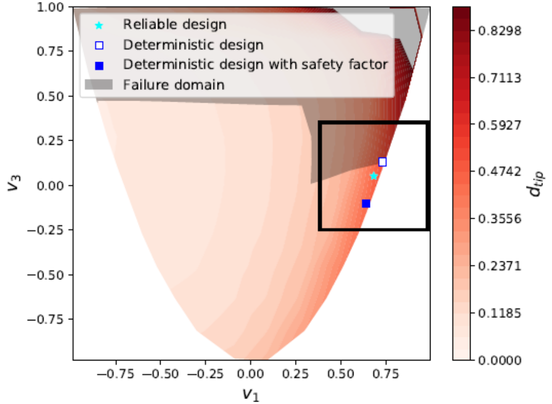
*Study cases and results*

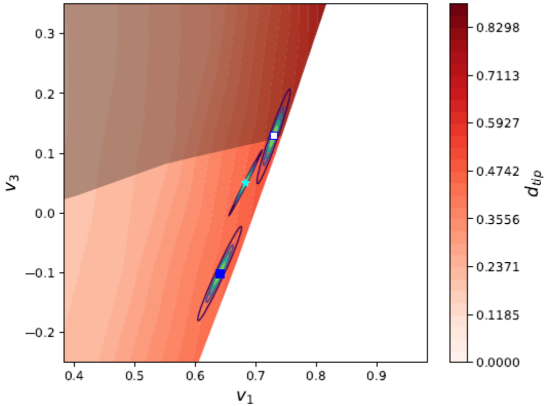
## Composite plate immersed in an airflow

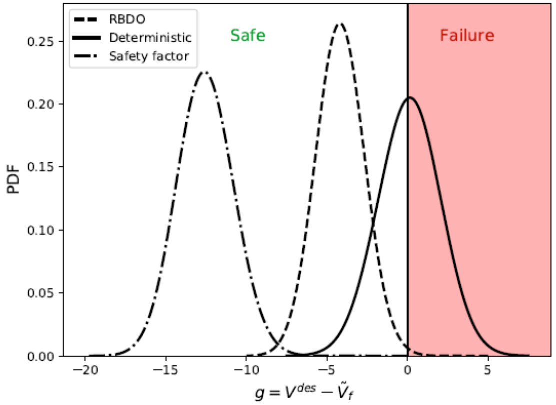
| Deterministic | Deterministic + safety factor | RBDO |
|---|---|---|
| $d_{tip} = 0{,}52$ | $d_{tip} = 0{,}41$ | $d_{tip} = 0{,}42$ |
| $P_f = 0{,}54$ | $P_f = 0$ | $P_f = 0{,}0005$ |



# Precedent studies – L.Coelho

## *Limitations and perspectives*

**Limitations:**

- Limited design space (no thickness)
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  - No sensitivity on the failure probability when optimizer travels into failure domain
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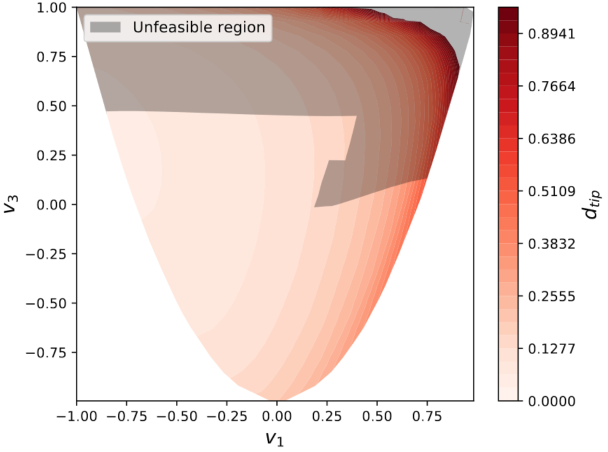
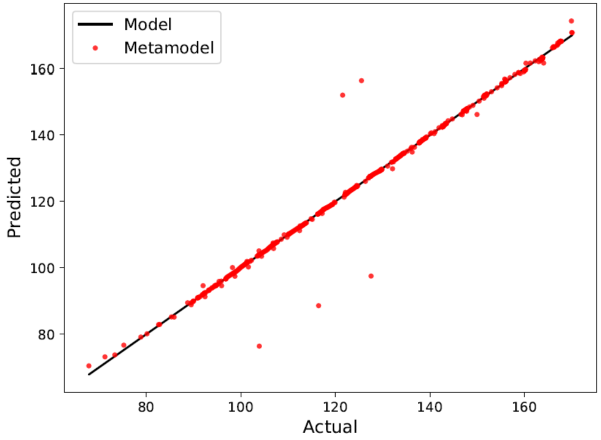


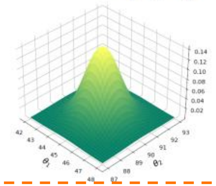



# Precedent studies – L.Coelho

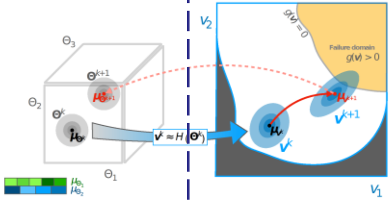
*Reliability Based Design Optimisation (RBDO)*

## Stacking sequence design space

$$\Theta = \mu_\Theta + \sigma_\Theta \boldsymbol{X}$$
$$X \sim \mathcal{N}(0,1)$$



## Lamination parameters design space




**Gradient optimiser**
Input: objective, constraints and gradients

**Design point**

**Inverse Problem**
Genetic algorithm used to compute corresponding stacking sequence

**Design laminate**

**Stacking error analysis**
Sample generation of layups with corresponding layup error

**Material Homogenisation**

**Failure probability analysis**
Analysis via Monte Carto method of the failure probability

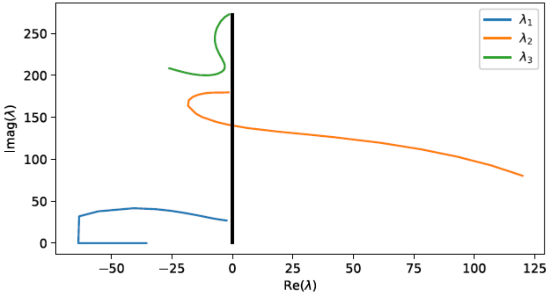
**Convergence**

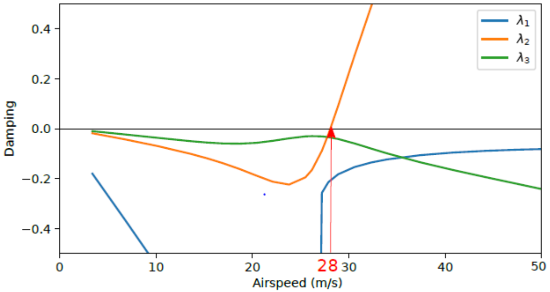
[1] Scarth & Cooper (2018) Composite Structures J.

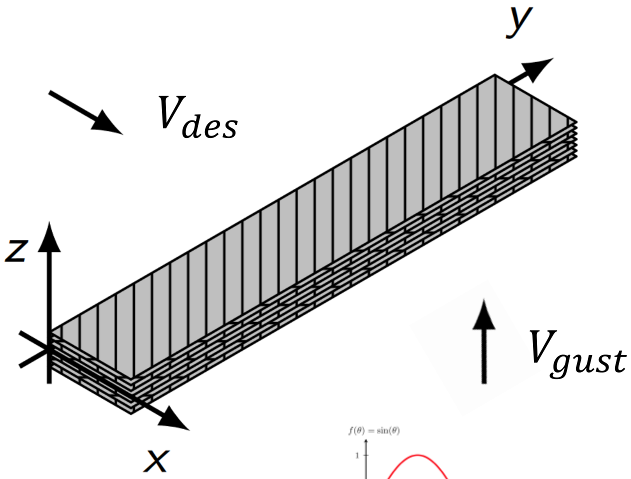
# Composite plate immersed in an airflow
## *Problem presentation*

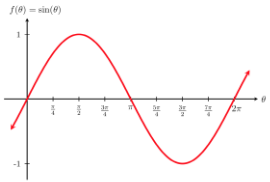
Three different analysis via NASTRAN (DLM) [1]

- **<u>Analysis 1:</u>** Fixed fluid velocity ($V_{des}$) and lift force ($L$)
  - → ***Output***: Tip displacement (mm) + strain(ε)
- **<u>Analysis 2:</u>** Flutter analysis
  - → ***Output***: Limit flutter speed (m/s)
- **<u>Analysis 3:</u>** Fixed fluid velocity ($V_{des}$) + vertical gust ($V_{gust}$)
  - → ***Output***: Maximum bending moment at base (Nm)



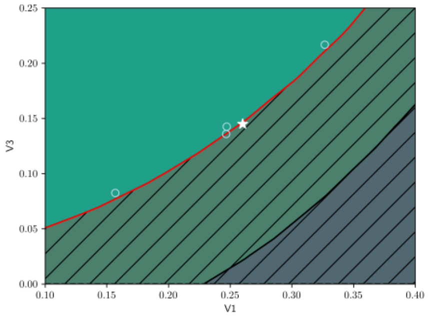



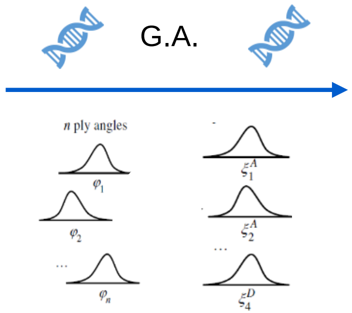



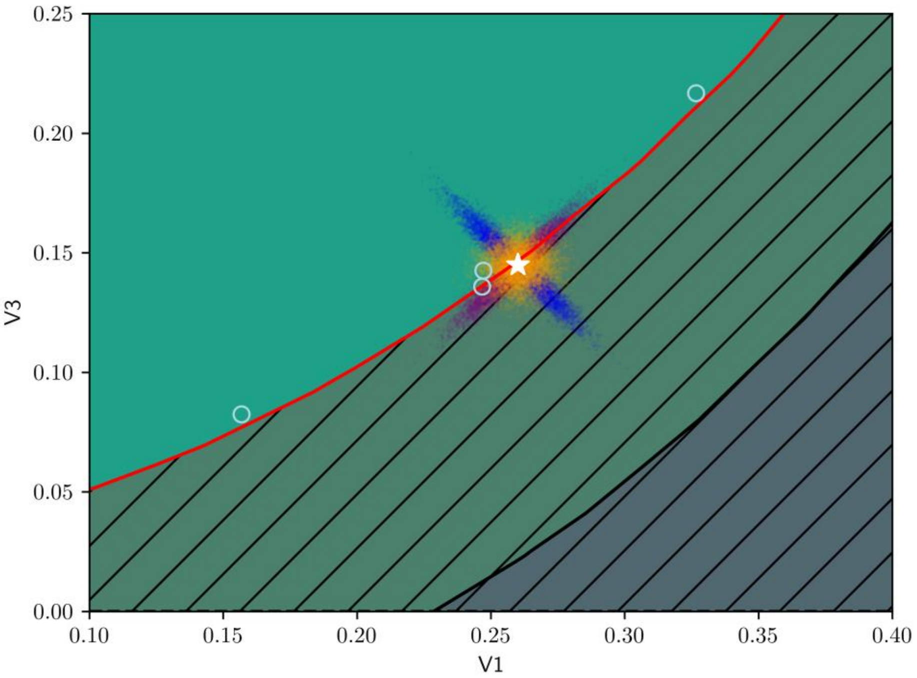
# Composite plate immersed in an airflow

*RBDO – Results-OLD*



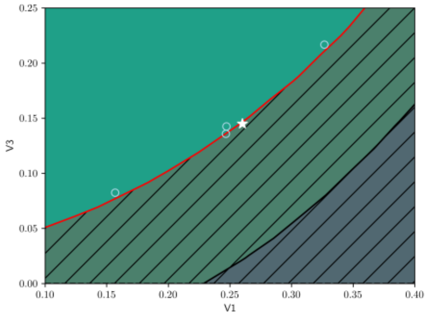

**Optimum value at:** $(V_D^1, V_D^3) = (0.2652, 0.1565)$

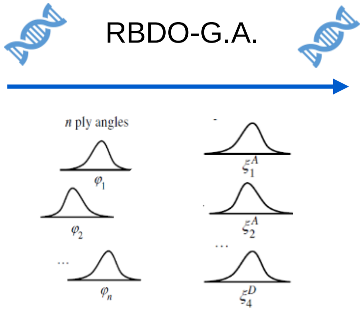



| | $\mu_{V1}$ | $\mu_{V3}$ | $F.P.$ |
|---|---|---|---|
| S1 | 0,2663 | 0,1587 | 0,732 |
| S2 | 0,2645 | 0,1538 | 0,623 |
| S3 | 0,2621 | 0,1567 | 0,445 |

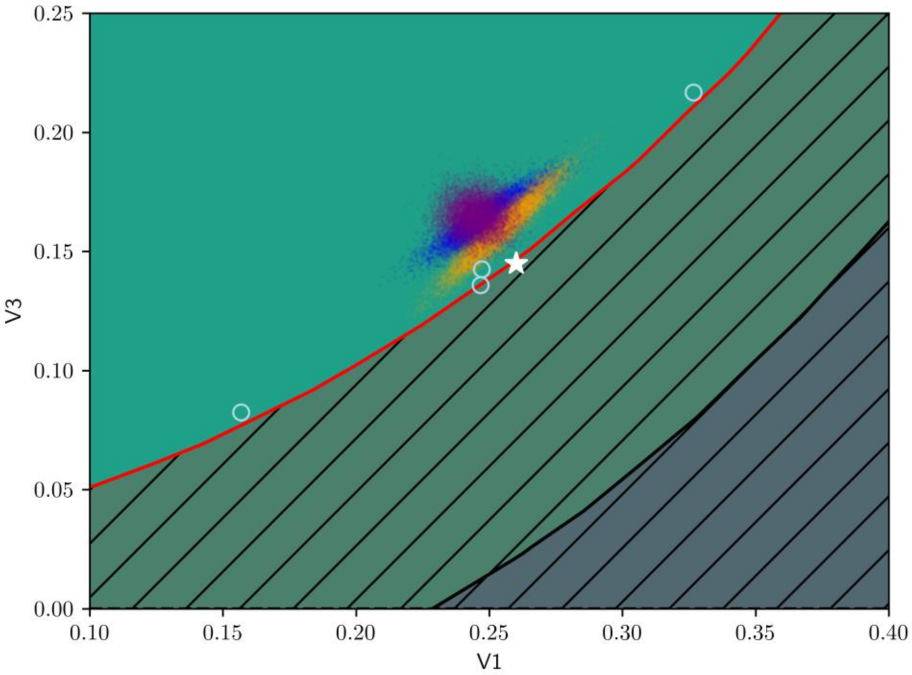
# Composite plate immersed in an airflow

*RBDO – Results - OLD*




**Optimum value at:** $(V_D^1, V_D^3) = (0.2652, 0.1565)$




| | $\mu_{V1}$ | $\mu_{V3}$ | $F.P.$ |
|---|---|---|---|
| S1 | 0,2663 | 0,1587 | 0,732 |
| S2 | 0,2645 | 0,1538 | 0,623 |
| S3 | 0,2621 | 0,1567 | 0,445 |
| S1 | 0,2456 | 0,1661 | 0,0005 |
| S2 | 0,2538 | 0,1604 | 0,0001 |
| S3 | 0,2505 | 0,1654 | 0 |